

OPTIMIZING PRODUCTION POLICIES FOR FLEXIBLE MANUFACTURING SYSTEM WITH NON-LINEAR HOLDING COST

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ABSTRACT

This paper deals with production inventory model for a deteriorating item over infinite planning horizon where the demand rate is a function of on hand inventory, production rate is a linear function of demand and thus non-linear function of on hand inventory. The traditional parameters of unit item cost and ordering cost are kept constant and comparative study of linear and non-linear holding cost is discussed to observe the effects of non-linearity of holding cost in flexible manufacturing system.

Keywords: Production policy, Non-linear holding cost, Flexible manufacturing system

INTRODUCTION

In the present consumerist society and a cut-throat competition in the market, the manufacturers are not only employing newer methods of distribution but also newer formats of distribution. The companies are entering rural markets, semi-urban areas and reaching out to the unexpected segments of potential customers. In addition to generating a spurt in demand, the companies are using innovative marketing strategies and innovative marketing tactics with varying degrees of effectiveness. As far as distribution is concerned, new departmental stores and new shopping malls are sprouting up even in the unrepresented geographical areas. As a result of all this, the visibility and reach of the brand/product has increased manifolds and it causes sudden fluctuations in demand. As such there is a strong need for a flexible manufacturing system that can take care of the above realities and adjusts itself to the realities of the market.

LITERATURE REVIEW

In the classical economic production lot size (EPLS) model, the manufacturing system is regarded to be inflexible. In recent years, flexibility in the production system has become an important aspect of the inventory models as they represent more realistic and practical situations. Advancement in the technology and instant communication has transformed the market mechanism and places tremendous pressure in making the availability of products instantaneously and timely. As per the emerging market trends, organizations tend to attract customers by displaying the items in large amount and in influential manner as we observe in big malls and stores. The attractive display of items in large amount and in more varieties influences and motivates the people to buy more. The tendency of buying more, results in greater demand and thus the demand is affected by the stock displayed. The changing trends in demand affect the production rate. To suit the recent market trends, researchers are being attracted towards volume flexible inventory system in which production rate is a decision variable and depends on various factors. Gupta and Vrat (1986) were

the first to develop models for stock dependent consumption rate. Mandal and Phaujdar (1989) developed an economic production quantity model for deteriorating items with constant production and stock dependent consumption rate. Datta and Paul (1990) focused on the analysis of the inventory system which describes the demand rate as a power function dependent on the level of on hand inventory and constant holding cost. Schweitzer, P.J. and Seidmann (1991) established optimizing processing rate for volume flexible manufacturing system. Khouja and Meraj (1995) extended the EPLS model with variable production rate and imperfect production. Giri and Chaudhary (1997) developed deterministic model of perishable inventory with stock dependent demand and non-linear holding cost. Wee (1999) developed the deteriorating inventory model with quantity discount, pricing and partial back-ordering. Mandal and Maiti (1999) considered the inventory of damageable items with variable replenishment rate and stock dependent demand and some units in hand. Skouri (2000) proposed an inventory model for deteriorating items with time varying demand and partial exponential type backlogging. Skouri and Papachristos (2003) enlightened the concept of optimal stopping and restarting production times for an economic order quantity model with deteriorating items and time dependent partial backlogging. Sana and Chaudhary (2004) developed a volume flexible production model for deteriorating items with time dependent demand, shortages, and stock dependent demand. Teng and Chang (2005) proposed economic production model for deteriorating items with price and stock dependent demand. Sana (2007) developed volume flexible inventory model with imperfect production process. Sana (2010) developed multi-item EOQ model of deteriorating items with demand influenced by enterprises initiatives. Sana (2011) extended an EOQ model for salesmen's initiative with stock and price sensitive demand. Sarkar and Moon (2011) developed an EPQ Model with inflation in an imperfect production system. Singh (2010) developed a supply chain model with stochastic lead time under imprecise partially backlogging and fuzzy ramp-type demand for expiring items. Singh (2010) contributed on an inventory model for deteriorating items with shortages and stock-dependent demand under inflation for two-shops under one management.

ASSUMPTIONS AND NOTATIONS

The following Assumptions and Notations are used to develop the Model:

ASSUMPTIONS

1. Production rate is linear function of demand.
2. The time horizon of the inventory system is infinite. Only a typical planning schedule of length T is considered, all remaining cycles are identical.
3. The demand rate is deterministic and is a known function of the on hand inventory q . The functional relationship between the demand rate $f(q)$ and the inventory level $q(t)$ is given by the following expression

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$$F(q) = Dq^\beta, D > 0, 0 < \beta < 1, q \geq 0$$

Where β denotes the shape parameter and is a measure of responsiveness of the demand to changes in the level of on hand inventory and D denotes the scale parameter.

4. A constant fraction θ , assumed small, of the on hand inventory gets deteriorated as per unit time.

NOTATIONS

$q(t)$:	on hand inventory level at any time t .
$f(q)$:	Demand rate, $f(q) = Dq^\beta, D > 0, 0 < \beta < 1$,
P	:	Production rate, $P = l f(q)$ where l is a scale parameter, $P > f(q), l > 1$
K	:	Ordering cost per order.
HC	:	Holding cost per cycle
DC	:	Deterioration cost per cycle.
TAC	:	Total average relevant inventory cost per unit time.
t_p	:	Time when inventory reaches its maximum.
c_p	:	Production cost per item.
h	:	Inventory holding cost per unit per unit time

MATHEMATICAL MODEL - A

At the beginning of each cycle, the inventory stock is zero and production starts at the very beginning of the cycle. As production continues inventory begins to pile up continuously after meeting demand and deterioration. Production stops at t_p . The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $[t_p, T]$. Each cycle consists of two stages:

The instantaneous states of $q(t)$ over the cycle time T is given by the following first order non-linear differential equations.

$$\frac{dq(t)}{dt} = (l-1)f(q) - \theta q(t) \quad 0 \leq t \leq t_p, \quad q(0) = 0 \quad (1)$$

$$\frac{dq(t)}{dt} + \theta q(t) = -f(q) \quad t_p \leq t \leq T, \quad q(t_p) = Q \quad (2)$$

Solving equation (1) & Using initial condition $q(0) = 0$

$$-\theta \alpha t = \log \left(1 - \frac{\theta q^\alpha}{(l-1)D} \right) \quad \text{Where } \alpha = 1 - \beta$$

On expansion of the right hand side, the first order approximation of θ gives

$$t = \left(\frac{q^\alpha}{(l-1)\alpha D} + \frac{\theta q^{2\alpha}}{2(l-1)^2 \alpha D^2} + \dots \right), 0 \leq t \leq t_p \quad (3)$$

Assume t_p be the time when inventory reaches its maximum level (say Q)

$$t_p = \frac{Q^\alpha}{(l-1)D\alpha} \left(1 + \frac{\theta Q^\alpha}{2(l-1)D} \right) \text{ Where } \alpha = 1 - \beta \quad 0 < \alpha < 1$$

Solving equation (2) & using condition $q(t_p) = Q$

$$q^\alpha e^{\theta \alpha t} = \frac{-D}{\theta} e^{\theta \alpha t} + \left(\frac{D}{\theta} + Q^\alpha \right) e^{\theta \alpha t_p}, t_p \leq t \leq T \quad (4)$$

Using $q(T) = 0$, the time to complete cycle is given by

$$T = t_p + \frac{Q^\alpha}{D\alpha} \left(1 - \frac{\theta}{2D} Q^\alpha \right) \text{ where } t_p = \frac{Q^\alpha}{(l-1)D\alpha} \left(1 + \frac{\theta Q^\alpha}{2(l-1)D} \right), \quad (5)$$

To make a comparative study, both cases: (i) linear holding cost and (ii) non linear holding cost, have been discussed.

CASE 1

When a holding cost is considered as linear function of on hand inventory:

$$\text{Holding cost in interval } [0, t_p] = h \int_0^{t_p} q dt \quad (6)$$

Making use of (3) in (6) and also the boundary conditions, we have

$$h \int_0^{t_p} q dt = \frac{h}{(l-1)D} \left[\frac{Q^{\alpha+1}}{\alpha+1} + \frac{\theta Q^{2\alpha+1}}{(2\alpha+1)(l-1)D} \right] \quad (7)$$

Similarly, finding holding cost in interval $[t_p, T]$

$$hc = h \int_{t_p}^T q dt$$

Using expansion, the first order approximation of θ in (4) gives

$$t = t_p + \frac{1}{\alpha} \left(\frac{Q^\alpha - q^\alpha}{D} - \frac{\theta}{2D^2} (Q^{2\alpha} - q^{2\alpha}) \right)$$

$$h \int_{t_p}^T q dt = h \left(\frac{Q^{\alpha+1}}{D(\alpha+1)} - \frac{\theta Q^{2\alpha+1}}{D^2(2\alpha+1)} \right) \quad (8)$$

Total holding cost in complete cycle $[0, T]$ using (7) & (8)

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$$HC = \frac{h}{(l-1)D} \left[\frac{Q^{\alpha+1}}{\alpha+1} + \frac{\theta Q^{2\alpha+1}}{(l-1)D(2\alpha+1)} \right] + h \left[\frac{Q^{\alpha+1}}{D(\alpha+1)} - \frac{\theta Q^{2\alpha+1}}{D^2(2\alpha+1)} \right] \quad (9)$$

Total deterioration cost in complete cycle $[0, T]$

$$DC = \theta c_p \left(\int_0^T q dt \right) = \theta c_p \left[\int_0^{tp} q dt + \int_{tp}^T q dt \right] \quad (10)$$

The total average inventory cost per unit time is, therefore given by

$$TAC = \frac{K + HC + DC}{T}$$

Our problem is to determine the time to stop the production when q takes its maximum value Q , which minimizes TAC of the inventory system. The necessary condition for TAC to be a minimum is

$$\frac{d}{dQ}(TAC) = 0$$

Which gives

$$T \left[\frac{d}{dQ}(HC) + \frac{d}{dQ}(DC) \right] - (K + HC + DC) \frac{dT}{dQ} = 0 \quad (11)$$

Substituting the values of T, HC & DC using (5), (9) & (10) in (11), we have

$$\left[tp + \frac{Q^\alpha}{D^\alpha} \left(1 - \frac{\theta}{2D} Q^\alpha \right) \right] \left[(h + \theta c_p) \left\{ \frac{1}{(l-1)D} \left(Q^\alpha + \frac{\theta Q^{2\alpha}}{(l-1)D} \right) + \left(\frac{Q^\alpha}{D} - \frac{\theta Q^{2\alpha}}{D^2} \right) \right\} \right] =$$

$$\left[K + (h + \theta c_p) \left\{ \frac{1}{(l-1)D} \left(\frac{Q^{\alpha+1}}{\alpha+1} + \frac{\theta Q^{2\alpha+1}}{(l-1)D(2\alpha+1)} \right) + \left(\frac{Q^{\alpha+1}}{D(\alpha+1)} - \frac{\theta Q^{2\alpha+1}}{D^2(2\alpha+1)} \right) \right\} \right] \frac{dT}{dQ}$$

Where $\frac{dT}{dQ} = \frac{Q^{\alpha-1}}{D} \left(\frac{l}{l-1} + \frac{\theta Q^\alpha}{2(l-1)^2 D^2} - \frac{\theta Q^\alpha}{2D} \right) + \frac{Q^\alpha}{D\alpha} \left(\frac{\theta\alpha Q^{\alpha-1}}{2(l-1)^2 D^2} - \frac{\theta\alpha Q^{\alpha-1}}{2D} \right)$

Considering special case as $\theta \rightarrow 0$

$$Q^{\alpha+1} = \frac{KD\alpha(\alpha+1)(l-1)}{hl} \quad \& \quad T = \frac{Q^\alpha l}{(l-1)D\alpha}$$

$$TAC = \frac{k + \frac{hlQ^{\alpha+1}}{(l-1)D(\alpha+1)}}{T}$$

Considering special case as $\theta \rightarrow 0, \alpha \rightarrow 1$, we have

$$Q^* = \sqrt{\frac{2KD(l-1)}{hl}}$$

As l increases, production occurs at a more rapid rate. Hence for large l , the model should approach the instantaneous delivery situation of the EOQ model.

For large l $1 - \frac{1}{l} \rightarrow 1$; Thus as l increases towards infinity; the optimal run size for the model approaches the EOQ.

CASE II

Non-linear stock dependent holding cost; when holding cost is considered as a power function of the on hand inventory:

$$\begin{aligned} HC &= \int_0^T hq^n dt \\ &= \int_0^{tp} hq^n dt + \int_{tp}^T hq^n dt \end{aligned} \tag{12}$$

Making use of (3) & (4) in (12)

$$\begin{aligned} &\text{Total holding cost in complete cycle } [0, T] \\ &= \frac{h}{(l-1)D} \left[\frac{Q^{n+\alpha}}{n+\alpha} + \frac{\theta Q^{n+2\alpha}}{(l-1)D(2\alpha+1)} \right] + h \left[\frac{Q^{n+\alpha}}{D(n+\alpha)} - \frac{\theta Q^{n+2\alpha}}{D^2(n+2\alpha)} \right] \\ &\& \quad Q^{*n+\alpha} = \frac{k(l-1)D(n+\alpha)\alpha}{nhl} \end{aligned}$$

As $n \rightarrow 1$ holding cost of case II approaches that of case I

Q^* and the corresponding value of cycle time T and TAC can be determined numerically.

Numerical example

In this part we present computational results obtained using Mathematica 7.0 which give insight about the behavior of optimal run size Q^* , production cycle time T and the total average cost TAC. The parameter values in both the models are taken as

$D=2.0$, $C=\$10.0$ per unit, $h=\$0.5$ per unit, $K=\$200$ per order, and $\theta=0.002$

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Table 1 presents the effects of the shape parameter β for various values of l on the approximate optimal solution and Table 2 presents the effects on optimal solution when holding cost is taken non linear.

Following observations are made from Table1:

- For a particular β , T generally decreases but Q and TAC increases as l increases.
- As β increases Q and TAC also increase but T decreases.

Following observations are made from Table1:

- For a particular β and particular n , T generally decreases but Q and TAC increases as l increases.
- For a particular β and particular l , Q and T decrease while TAC increases as the degree of non linearity in the holding cost increases.

Comparative observations:

- TAC is much higher when the nonlinear stock-dependent holding cost is included in the inventory model.

Q and T are much less in the inventory system with non linear holding cost as compared to inventory system with linear holding cost.

TABLE 1: Effects of $\alpha = 1 - \beta$ and l on (Q, T, TAC) for case I

l/α		0.9	0.7	0.5	0.3
2	Q	31.0552	37.5892	44.814	48.64
	T	24.4726	18.09	13.3887	10.6902
	TAC	15.5272	18.79	22.407	24.32
4	Q	38.44	47.714	58.723	66.4467
	T	19.76	14.25	10.2	7.8258
	TAC	19.22	23.85	29.36	33.2234
6	Q	40.635	50.76	62.99	72.056
	T	18.703	13.39	9.52	4.695
	TAC	20.317	25.38	31.49	55.3723

TABLE 2: Effects of $\alpha = 1 - \beta$, l and n on (Q, T, TAC) for case II

$l = 2$					
n/α		0.9	0.7	0.5	0.3
2	Q	8.652	9.008	9.1028	8.519
	T	7.747	6.655	6.034	6.34
	TAC	37.4305	40.57	41.431	36.29
4	Q	3.4648	3.432	2.945	3.0962
	T	3.3997	3.386	3.43	4.678
	TAC	72.0602	69.38	62.45	45.95
$l = 4$					
n/α		0.9	0.7	0.5	0.3
2	Q	9.9506	10.4676	10.705	3.4024
	T	5.8577	4.93	4.36	3.208
4	TAC	49.5071	54	57	67.01
	Q	3.763	4.336	3.646	10.16

	T	2.4418	2.659	2.54	4.4
	TAC	100.33	101.52	88.374	51.62

MODEL B: Shortages are allowed.

Along with the assumptions in Model A, if we add one more assumption that shortages are allowed. Each cycle now consists of four stages. The initial stock in each cycle is zero and production starts at the very beginning of the cycle. As production continuous, inventory begins to build up continuously after meeting demand and deterioration. Production is stopped at a certain time. The accumulated inventory is consumed and ultimately becomes zero due to demand and deterioration. Production does not restart at this stage and inventory shortages continue to accumulate for some time. Thereafter, production starts and shortages are gradually cleared after meeting current demands. The cycle ends with zero inventories.

NOTATIONS: Along with previous notations

- S : Maximum shortage level
- t_p : time when inventory reaches its maximum level
- SC : Shortage cost per cycle
- t_s : time when shortages start
- t_m : time when maximum shortages occur
- T : Time of completion of cycle

Mathematical model B: Forward manufacturing system

The four stages are represented by four differential equations below:

$$(1b) \quad \frac{dq}{dt} + \theta q = (l-1)Dq^\beta, \quad q(0) = 0, \quad 0 \leq t \leq t_p$$

$$(2b) \quad \frac{dq}{dt} + \theta q(t) = -Dq^\beta, \quad q(t_p) = Q, \quad t_p \leq t \leq t_s$$

$$(3b) \quad \frac{dq}{dt} = -Dq^\beta, \quad q(t_s) = 0, \quad t_s \leq t \leq t_m$$

$$(4b) \quad \frac{dq}{dt} + \theta q(t) = (l-1)Dq^\beta, \quad q(t_m) = S, \quad t_m \leq t \leq T$$

(1b) and (2b) has already been solved in Model A. solving (3b) & (4b) and using boundary conditions, the four solutions are as below:

$$(1bs) \quad t = \left(\frac{q^\alpha}{(l-1)\alpha D} + \frac{\theta q^{2\alpha}}{2(l-1)^2 \alpha D^2} + \dots \right), \quad 0 \leq t \leq t_p$$

$$(2bs) \quad q^\alpha e^{\theta \alpha t} = \frac{-D}{\theta} e^{\theta \alpha t} + \left(\frac{D}{\theta} + Q^\alpha \right) e^{\theta \alpha t_p}, \quad t_p \leq t \leq t_s$$

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$$(3bs) \quad \frac{q^\alpha}{\alpha} = D(t_s - t), \quad t_s \leq t \leq t_m$$

$$(4bs) \quad q^\alpha = \frac{(l-1)D}{\theta} + \left(S^\alpha - \frac{(l-1)D}{\theta} \right) e^{\theta\alpha(t_m-t)}, \quad t_m \leq t \leq T$$

The cycle consists of four stages; time for each stage and the cycle time have been calculated in the same manner as in model A and are as below:

t_p is the time when production is stopped & inventory reaches its maximum.

$$t_p = \frac{Q^\alpha}{(l-1)D\alpha} \left(1 + \frac{\theta Q^\alpha}{2(l-1)D} \right)$$

t_s is the time when shortages start & accumulated inventory level is nil.

$$t_s = t_p + \frac{Q^\alpha}{D\alpha} \left(1 - \frac{\theta Q^\alpha}{2D} \right)$$

t_m is the time when maximum shortages occur & again production is started;

$$t_m = t_s - \frac{S^\alpha}{D\alpha}$$

$$T = t_m + \frac{S^\alpha}{(1-l)D\alpha} \left(1 - \frac{\theta S^\alpha}{2(1-l)D} \right)$$

Which represents complete cycle time.

Now we calculate the various associated costs, holding cost for the new inventory model remains same as it was in model A as there is no inventory during the new added period of shortage $[t_s, T]$.

To find cost of shortage in interval $[t_s, t_m]$

Using (3bs) and boundary conditions, we obtain shortage cost in $[t_s, t_m]$

$$\begin{aligned} sc &= -c_S \int_{t_s}^{t_m} q dt \\ &= \frac{c_S}{D} \left(\frac{S^{\alpha+1}}{\alpha+1} \right) \end{aligned} \quad (13)$$

Again production starts at $t = t_m$ and shortages are completely backlogged after meeting the current demand and deterioration.

$$\begin{aligned} \text{Shortage cost in interval } [t_m, T] &= -c_s \int_{t_m}^T q dt \\ &= c_s \left(\frac{S^{\alpha+1}}{(l-1)D(\alpha+1)} + \frac{\theta S^{2\alpha+1}}{(l-1)^2 D^2(2\alpha+1)} \right) \end{aligned} \quad (14)$$

Adding (13) & (14) the total shortage cost in the complete cycle $[0, T]$ are as below

$$SC = \frac{c_s S^{\alpha+1} l}{D(l-1)(\alpha+1)} + \frac{\theta c_s S^{2\alpha+1}}{(l-1)^2 D^2(2\alpha+1)} \quad (15)$$

Deterioration cost in $[t_m, T]$

$$\begin{aligned} &= \theta c_p \int_{t_m}^T q dt \\ &= -\theta c_p \left(\frac{S^{\alpha+1}}{(l-1)D(\alpha+1)} + \frac{\theta S^{2\alpha+1}}{(l-1)^2 D^2(2\alpha+1)} \right) \end{aligned}$$

The total inventory cost per unit time is therefore given by

$$TAC(Q, S) = \frac{K + HC + DC + SC}{T}$$

TAC (Q,S) is a function of Q & S & our problem is to find the time to stop the production when q takes maximum value Q and to find the time to again start the production when maximum shortages accumulate. The necessary conditions for extreme values of TAC (Q, S) are:

$$\frac{\partial}{\partial Q}(TAC) = 0, \frac{\partial}{\partial S}(TAC) = 0$$

$$\text{Using } \frac{\partial}{\partial S}(TCU) = 0$$

$$T \left[\frac{\partial}{\partial S}(SC) + \frac{\partial}{\partial S}(DC) \right] - (K + HC + DC + SC) \frac{\partial T}{\partial S} = 0 \quad (16)$$

Where,

$$T = \frac{Q^\alpha}{(l-1)D\alpha} \left(1 + \frac{Q^\alpha \theta}{2(l-1)D} \right) + \frac{Q^\alpha}{D\alpha} \left(1 - \frac{\theta Q^\alpha}{2D} \right) - \frac{S^\alpha}{D\alpha} + \frac{S^\alpha}{(l-1)D\alpha} \left(1 - \frac{\theta S^\alpha}{2(1-l)D} \right)$$

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Substituting all the relevant values in (16), we have

$$\begin{aligned} & \left[K + (h + \theta c_p) \left\{ \frac{1}{(l-1)D} \left(\frac{Q^{\alpha+1}}{\alpha+1} + \frac{\theta Q^{2\alpha+1}}{(l-1)D(2\alpha+1)} \right) + \left(\frac{Q^{\alpha+1}}{D(\alpha+1)} - \frac{\theta Q^{2\alpha+1}}{D^2(2\alpha+1)} \right) \right\} \right. \\ & \quad + c_s \left\{ \frac{S^{\alpha+1}l}{D(l-1)(\alpha+1)} + \frac{\theta S^{2\alpha+1}}{(l-1)^2 D^2(2\alpha+1)} \right\} \\ & \quad \left. - \theta c_p \left\{ \frac{s^{\alpha+1}}{(l-1)D(\alpha+1)} + \frac{\theta S^{2\alpha+1}}{(l-1)^2 D^2(2\alpha+1)} \right\} \right] \\ & \quad \times \left[\frac{-S^{\alpha-1}}{D} + \frac{S^{\alpha-1}}{(1-l)D} \left(1 - \frac{\theta S^\alpha}{2(1-l)D} \right) + \frac{S^\alpha}{(1-l)D\alpha} \left(\frac{-\theta\alpha S^{\alpha-1}}{2(1-l)D} \right) \right] \\ & = \left[\frac{Q^\alpha}{(l-1)D\alpha} \left(1 + \frac{Q^\alpha \theta\alpha}{2(1-l)D\alpha} \right) + \frac{Q^\alpha}{D\alpha} \left(1 - \frac{\theta Q^\alpha}{2D} \right) - \frac{S^\alpha}{D\alpha} + \frac{S^\alpha}{(1-l)D\alpha} \left(1 - \frac{\theta}{2(1-l)D} \right) \right] \\ & \quad \times \left[\left(\frac{c_s S^\alpha}{D} + \frac{c_s S^\alpha}{(l-1)D} + \frac{\theta c_s S^{2\alpha}}{(l-1)^2 D^2} \right) - \theta c_p \left(\frac{S^\alpha}{(l-1)D} + \frac{\theta S^{2\alpha}}{(l-1)^2 D^2} \right) \right] \end{aligned}$$

Considering the special case when $\theta \rightarrow 0, \alpha \rightarrow 1$ and substituting all the relevant values in (16), we have

$$(Q - S)(c_s S) \left(\frac{l}{l-1} \right) = \left(K + \frac{hQ^2}{2D} \frac{l}{l-1} + \frac{c_s S^2 l}{2D(l-1)} \right)$$

Now using $\frac{\partial TAC}{\partial Q} = 0$

$$\tau \left(\frac{\partial(HC)}{\partial Q} + \frac{\partial}{\partial Q}(DC) \right) = (K + HC + DC + SC) \frac{\partial T}{\partial Q} \quad (17)$$

Dividing (16) & (17) and $\theta \rightarrow 0, \alpha \rightarrow 1$, we get a relation in S and Q

$$c_s S = -hQ$$

Substituting $c_s S = -hQ$ in (16)

$$Q^* \rightarrow \sqrt{\frac{2KD}{h} \left(1 - \frac{1}{l} \right) \frac{c_s}{h + c_s}}$$

As $l \rightarrow \infty$

$$Q^* \rightarrow \sqrt{\frac{2KDc_s}{h(h+c_s)}}$$

Which is result obtained in EOQ model when shortages are allowed.

CONCLUSION

In this article, an inventory model with flexible manufacturing system, stock dependent demand rate is developed for an infinite planning horizon. In Model A, both cases (i) linear holding cost (ii) non-linear holding cost have been discussed and optimal solutions are derived for both the cases. In particular, a numerical example has been presented to discuss the effect of non-linear holding cost. It is observed that total inventory cost is much high when the non-linear stock dependent holding cost is encountered in the inventory system. In Model B, shortages are allowed and optimal solution is derived. The proposed article can be extended by including reverse manufacturing, inflation and discounting and other assumptions.

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